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Calculation of O_4 weighting functions

$\mathrm{SCD}_{\mathrm{O_4}}$ resp. $\tau_{\mathrm{O_4}}$ at one wavelength λ:

$$\tau_{\mathrm{O_4}} = \sigma_{\mathrm{O_4}} \mathrm{SCD}_{\mathrm{O_4}} = \sigma_{\mathrm{O_4}} \sum_{c=1}^{N} l_c n_c = \sum_{c=1}^{N} l_c \beta_{c,\mathrm{O_4}} \tag{1}$$

l_c sensitivity, n_c number concentration of O4 in the cluster c. There are N clusters. l_c can be expressed as:

$$l_c = -\frac{d \log(I)}{d\beta_{c,\mathrm{O_4}}} = -\frac{1}{I} \frac{dI}{d\beta_{c,\mathrm{O_4}}}. \tag{2}$$

O_4 weighting functions:

$$\frac{d\tau_{\mathrm{O_4}}}{dx_{c'}} = -\frac{d}{dx_{c'}} \sum_{c=1}^{N} \frac{\beta_{c,\mathrm{O_4}}}{I} \frac{dI}{d\beta_{c,\mathrm{O_4}}} = \frac{1}{I^2} \sum_{c=1}^{N} \beta_{c,\mathrm{O_4}} \left(\frac{dI}{d\beta_{c,\mathrm{O_4}}} \frac{dI}{dx_{c'}} - I \frac{d^2 I}{d\beta_{c,\mathrm{O_4}} dx_{c'}} \right). \tag{3}$$

Therefore one needs the radiance I, the Jacobians $\frac{dI}{d\beta_{c,\mathrm{O_4}}}$ and $\frac{dI}{dx_{c'}}$ and the Hesse matrix elements $\frac{d^2 I}{d\beta_{c,\mathrm{O_4}} dx_{c'}}$ for the calculation of O_4 weighting functions.